Tallinn University of Technology
Department of Cybernetics, School of Science

Research Report 333/26

Annual Report 2025
on Nonlinear Dynamics and Biophysics

Jüri Engelbrecht
Tanel Peets
Kert Tamm
Muhammad Zeeshan Ashraf

Tallinn, 2026

Contents

1. Introduction

This Report continues the series of Annual Reports on nonlinear dynamics started in 1999 within the Institute of Cybernetics. After restructuring of the Tallinn University of Technology, the studies on nonlinear waves are carried on in the Department of Cybernetics, the School of Science. What follows, is the description of the results of the subgroup of biophysical modelling in the Laboratory of Solid Mechanics during 2025. The attention is focused on dynamical processes and waves in nerve fibres which means working at the interface of physics, mathematics and physiology.

2. Research results

Waves in nerve fibres

Papers

1. J.Engelbrecht, K.Tamm, T. Peets. Interdisciplinarity in modelling of biophysical processes. Proc. Estonian Acad. Sci., 2025, 74, 544-550. https://doi.org/10.3176/proc.2025.4.08

Abstract: In this essay, the importance of interdisciplinary ideas is described for modelling and understanding signal propagation in nerve fibres as a fascinating problem of biophysics. A mathematical model involves the physical laws, assumptions, hypotheses, and finally, the governing equations. The analysis of this complex process is at the interface of physics, chemistry, and mathematics, together with experimental studies in electrophysiology. It is stressed that the mindsets of different communities may hinder cooperation.

2. K. Tamm, T.Peets, J.Engelbrecht. The modelling of the action potentials in myelinated nerve fibres. ArXiv:2406.18590v3 [physics.bio-ph] 1.July 2025
BMMB (accepted), https://doi.org/10.1007/s10237-025-02030-w

Abstract: The classical Hodgkin–Huxley model describes the propagation of an action potential (AP) in unmyelinated axons. In many cases, the axons have a myelin sheath and the experimental studies have then revealed significant changes in the velocity of APs. In this paper, a theoretical model is proposed describing the AP propagation in myelinated axons. As far as the velocity of an AP is affected, the basis of the model is taken after Lieberstein, who included the possible effect of inductance that might influence velocity, into the governing equation. The proposed model includes the structural properties of the myelin sheath: the μ-ratio (the ratio of the length of the myelin sheath and the node of Ranvier) and g-ratio (the ratio of the inner-to-outer diameter of a myelinated axon) through parameterγ. The Lieberstein model can describe all the essential effects characteristic to the formation and propagation of an AP in an unmyelinated axon. Then a phenomenological model (a wave-type equation) for a myelinated axon is described including the influence of the structural properties of the myelin sheath and the radius of an axon. The numerical simulation using the physical variables demonstrates the changes in the velocity of an AP. These results match well the known effects from experimental studies.

Papers submitted

1. K.Tamm, T.Peets, J.Engelbrecht. On hyperbolicity for nerve pulse propagation in axons. Journal of Computational Neuroscience.

   Abstract: The classical Hodgkin-Huxley (HH) model describes the propagation of an action potential (AP) in unmyelinated axons. Here, the key difference from the classical HH model is the hypothesis that the AP propagation in axons depends not only on the HH ion mechanism but also on capacitance and inductance. The modified model is hyperbolic (wave-equation type) compared to the classical HH model, which is parabolic (diffusion-equation type). In this paper, we revisit a modification of the classical HH model proposed by Lieberstein for describing propagating AP in an unmyelinated axon, including the possible effect of inductance that might influence velocity, into the governing equation. We briefly revisit and discuss the underlying assumptions about the hypothesis of including the inductivity in the HH model, systematically check the behaviour of the solutions of the modified governing equations to make sure it is correct for describing the AP and finally discuss some aspects which are relevant for the further modifications (like, for example, taking into account the effect of myelination). The numerical simulation using the physical variables demonstrates the changes in the velocity of an AP as well as the changes in its profile. These results match well the known effects from experimental studies.

2. M.Z.Ashraf, T.Peets, K.Tamm, A charge conservative finite volume discretization of the Hodgkin-Huxley model. Journal of Computational Neuroscience.

   Abstract: This article presents a computational study of Hodgkin-Huxley model and the simulation of action potential propagation in an unmyelinated axon using finite volume method. By implementing a voltage-clamp patch and tracking detailed ionic and capacitive currents and channel gating we achieve robust and unit-consistent simulation of action potential initiation and propagation.

Papers in preparation
1. J.Engelbrecht, K.Tamm, T.Peets. From modelling to understanding: the signals in nerves.

Books
1. J.Engelbrecht. Puuduvad leheküljed. Postimehe kirjastus, Tallinn, 2025.
(in Estonian: Missing Pages, Postimees Publ., Tallinn, 2025)
2. Ü. Mander, J.Engelbrecht (koost., toim.) Teadusmõte Eestis (XI). Globaalprobleemid. Eesti Teaduste Akadeemia, Tallinn, 2025.
(in Estonian: Science in Estonia XI. Global Problems. Estonian Academy of Sciences, Tallinn, 2025)

General articles
1. J.Engelbrecht. Teadmistest globaalses maailmas. Mente et manu. 1(1902), 2025, 18-21.
2. Ü. Mander, J. Engelbrecht. Saateks. Kogumik "Teadusmõte Eestis (XI). Globaalprobleemid" . Tallinn, Eesti Teaduste Akadeemia, 2025, 7-9.
3. J.Engelbrecht. Kompleksssüsteemid ja globaalsed probleemid. Kogumik "Teadusmõte Eestis (XI). Globaalprobleemid". Tallinn, Eesti Teaduste Akadeemia, 2025, 10-19.
4. J.Engelbrecht. Teerajaja ja innustaja. Väitlus:
Taivo Liiva. Mälestuskilde Nikolai Alumäest ja tema kaasvõitlejaist ehk kui mu ees paotus uks teadusmaailma, Akadeemia, 2025, vol. 37, No 4, lk 641-659; No 5, lk 855-903.
Akadeemia, 2025, vol. 37, no 9, 1697-1704.
5. J.Engelbrecht. Laual olev probleem on kõige huvitavam. Mente et Manu. Eriväljaanne, August, 2025, 18-19.

General articles (submitted)
1.. J.Engelbrecht, T.Tiivel, H.Lippmaa. Akadeemiline Inglise klubi (Academic English Club) 1961-1995 - omapärane nähtus Eesti teadus-jakultuurielus. Akadeemia (ilmumas 2026).
2. J.Engelbrecht. Sõnade vägi mõtteloos (lühendatud tekst). Conference "The Power of Words - the Half a Century since the Founding of the Baltic Union by Estophil Vladimir Macura. Proceedings (to be published in 2026).

Research Reports
1.J.Engelbrecht, K.Tamm, T.Peets. Annual Report 2024 on Nonlinear Dynamics and Biophysics. RR Mech 332/25, Tallinn University of Technology, School of Science, Dept. of Cybernetics.

Conferences

1.J.Engelbrecht. The power of words in the story of thoughts by H.Runnel. Conference "The Power of Words - the Half a Century since the Founding of the Baltic Union by Estophil Vladimir Macura. Prague, Jan., 9, 2025

2. K.Tamm, T.Peets, J.Engelbrecht. The modelling of the action potentials in myelinated nerve fibres. CNS 2025, 34th Annual Neuroscience Meeting, July 5-9, 2025, Florence.

3. T.Peets, K.Tamm, J.Engelbrecht. Modelling of ensemble of signal in single axons. CNS 2025, 34th Annual Neuroscience Meeting, July 5-9, 2025, Florence.

4. J.Engelbrecht. Lõppsõna. Konverents " Vladimir Macura Praha ja Tallinna vahel", Tallinn 16. sept, 2025. Eesti Teaduste Akadeemia ja Tšehhi Teaduste Akadeemia (Tšehhi Vabariigi Suursaatkonna egiidi all)

3.Activities

Teaching:

K. Tamm - Courses in Tallinn University of Technology:
BSc level: Fundamentals of Elasticity (YFX0592),
Analytical Mechanics (YFX0591)

T. Peets - Courses in Tallinn University of Technology:
BSc level: Writing Academic Papers and Theses (YFX0540),
Mechanics (YFX0552)
MSc level: Mathematical Modelling (YFX1570)

Supervision of theses:

BSc - Artur Leppik - Termomehaaniliste protsesside modelleerimine biomembraanis.
Supervised by K.Tamm and T.Peets
BSc - Raigo Milvaste - Modeling Action Potential Propagation in Unmyelinated Axons.
Supervised by K.Tamm and T.Peets

Reviewing:

K.Tamm –
[1] MSc väitekiri “Difusiooniuuringud südame lihasrakkudes”, autor Otto Gustavson,
[2] review for Journal of Neural Engineering.
T. Peets – review for Plos Computational Biology.
J.Engelbrecht - MSc väitekiri "Kaltsiumi aktivatsioon südamelihases. matemaatiline mudel ja eksperimentaalsed katsed", autor Maike Mona Širinov.

Membership in Editorial Boards:

Applied and Computational Mechanics (Czech Republic): J.Engelbrecht;
Proc. Estonian Acad. Sci.: J.Engelbrecht;
Applied Mechanics (Kiev): J.Engelbrecht;
J.Theor. and Appl. Mech. (Warsaw): J.Engelbrecht;
Trames (Estonia): J.Engelbrecht;
Akadeemia (Estonia): J.Engelbrecht.

Professional organizations:
Euromech: J.Engelbrecht, K.Tamm, T.Peets;
Euromembrane: K.Tamm, T.Peets;
Organization for Computational Neurosciences: K.Tamm, T.Peets;
ISIMM: J.Engelbrecht;
Nordic Association for Computational Mechanics: T.Peets (member of the Executive Committee);
Estonian Academy of Sciences: J.Engelbrecht (Fellow, Adviser);
WAAS – World Academy of Art and Science: J.Engelbrecht (Fellow);
Estonian National Committee for Mechanics: T.Peets (secretary), J.Engelbrecht (member).

4. Grants and cooperation

1. T. Elmqvist, ..., J.Engelbrecht, ... (team of 23 members). Changing Wildfires. Policy Options for a Fire-Literate and Fire-Adapted Europe. EASAC Report 48, May 2025 (launched May 19, 2025, Brussels).

5. Varia

1.J. Engelbrecht. TTÜ aastavilistlase 2024 tervitus TTÜ 2025/26 avaaktusel.
2. J.Engelbrecht. TTÜ Honoris Universitatis 2025.

6. Conclusions

The research on modification of the model (described in a book by Springer in 2021) is continued with the main attention to the refining of the model by taking the structural details (the geometry of myelination) of nerve fibres into account. Attention is paid to interdisciplinarity needed for modelling the processes in nerve fibres. several papers are in preparation. The method of finite volume is applied in the numerical simulation of the AP propagation.

Concerning the previous results:
- papers by J .Engelbrecht have more than 20 000 "reads" in ResearchGate;
- the book "Microstructured Materials: Inverse Problems" (Springer, 2011) - has ca 8500 downloads;
- the book "Questions About Elastic Waves"(Springer, 2015) - has ca 9000 downloads;
- the book "Modelling of Complex Signals in Nerves" (Springer, 2021) has ca 3700 downloads;
- the book "Applied Wave Mathematics II" (Springer 2019) has ca 11000 chapter downloads and the chapter "Mathematics of nerve signals" - over 800 downloads.

The recent results are widely known according to SciVal lists on topics by authors over the world (in brackets the place in the list):
- Thermodynamic Insights into Nerve Action Potential:
J.Engelbrecht (1), T.Peets (2), K.Tamm (3).

E-Mails:
Jüri Engelbrecht: je@ioc.ee
Tanel Peets: tanelp@ioc.ee
Kert Tamm: kert@ioc.ee
Webpage: https://solmech.taltech.ee